Magnetostatic Waves in a Normally Magnetized Waveguide Structure

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Abstract —In this paper, the propagation of magnetostatic waves (MSW's) in a normally magnetized low-loss ferrite slab (such as a yttrium iron garnet (YIG) slab) placed inside a waveguide is investigated theoretically. This case has never been studied before, and is analyzed here for the first time.

A dispersion relation for the modes of propagation in terms of an infinite determinant can be obtained. With proper truncation procedures, sample numerical calculations for dispersion relations and group time delay per unit length were obtained and are presented herein. The general formulation in this paper contains all the information provided by the degenerate cases previously published. One special case of interest, i.e., that of a multilayer planar structure, is derived from our general formulation. The derivations of other special cases follow the same procedure.

I. Introduction

MAGNETOSTATIC WAVE propagation in a ferrite slab completely filling a waveguide or otherwise bounded by metallic walls has been reported in the literature [1]–[3]. Recently the analysis of magnetostatic wave propagation in a partially YIG-loaded waveguide was reported [4], [5]. In these recent developments, the direction of the dc magnetic field was assumed to be parallel to the slab and perpendicular to the direction of wave propagation which led to the propagation of magnetostatic surface waves (MSSW's). These waves are highly nonreciprocal with regard to the direction of propagation and unsymmetrical with respect to the slab position in the waveguide.

The case of a normally magnetized YIG slab partially filling a waveguide has never been approached and remains yet unsolved.

In this paper, the dc magnetic field is perpendicular to the slab plane (see Fig. 1). This leads to the propagation of magnetostatic forward volume waves (MSFVW's), which are reciprocal and symmetrical.

An analytical expression for the dispersion relation is derived in Section II. Some numerical computations for the dispersion relations and group time delay for certain values of the dc magnetic field were obtained and the results of this simulation are presented in Section III. Also in this section, one special case, i.e., that of a multilayer planar structure, is derived from the formulation developed in this paper. Final conclusions and some discussions are given in Section IV.

Manuscript received March 12, 1987; revised July 2, 1987. This work was supported in part by the Air Force Systems Command, Avionics Laboratory, Wright-Patterson Air Force Base, OH, under Contract F-33615-81-8-1429.

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IEEE Log Number 8717119.

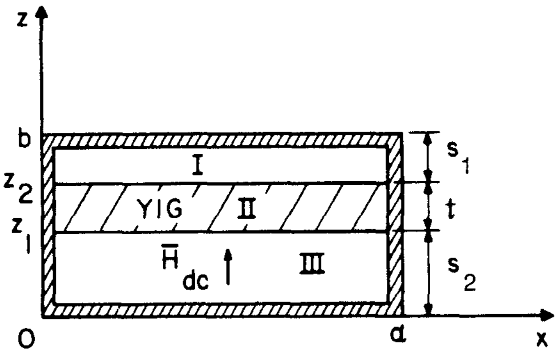

Fig. 1. Partially loaded waveguide with H_{dc} normal to the YIG slab.

II. Analysis

The relative permeability tensor when $\overline{H}_{dc} = H_0\hat{z}$, can be expressed as

$$\bar{\bar{\mu}}_r = \begin{bmatrix} \mu & jK_1 & 0 \\ -jK_1 & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where

$$\mu = 1 + \frac{\omega_0 \omega_M}{\omega_0^2 - \omega^2}$$

$$K_1 = \frac{\omega \omega_M}{\omega_0^2 - \omega^2}$$

$$\omega_0 = \mu_0 \gamma H_0$$

$$\omega_M = \mu_0 \gamma M_0.$$

Here μ_0 and γ are the free-space permeability constant and gyromagnetic constant (2.8 MHz/Oe), respectively, ω is the operating frequency, and H_0 and M_0, in oersteds, are the internal magnetic field and saturation magnetization [6], respectively (1 Oe $= 1000/4\pi$ A/m).

For simplicity of analysis, we assume that the demagnetizing fields are negligible. In this case the external magnetic field (H_{dc}) becomes equals to the internal magnetic field, i.e., $H_{dc} = H_o$.

For magnetostatic waves, the small-signal magnetic field intensity ($\bar{h}$) and the small-signal magnetic flux density ($\bar{b}$) are given by

$$\bar{h} = \nabla\phi \tag{1}$$

$$\nabla \cdot \bar{b} = 0 \tag{2}$$

$$\bar{b} = \mu_0 \bar{h} \qquad \text{(in air)} \tag{3}$$

$$\bar{b} = \mu_0 \bar{\bar{\mu}}_r \bar{h} \qquad \text{(in ferrite)} \tag{4}$$

where ϕ is the scalar magnetic potential satisfying (from (1),(2),(3),(4))

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{5}$$

in the air regions (I and III) and

$$\mu(\phi_{xx} + \phi_{yy}) + \phi_{zz} = 0 \tag{6}$$

in the ferrite region.

The implied time dependence is $e^{j\omega t}$ and is omitted in all of the following expressions. The variation of ϕ in the axial direction (y) is assumed to be of the form e^{-jKy}, where K is the wavenumber. Therefore (5) and (6) can be rewritten as

$$\phi_{xx} + \phi_{zz} = K^2\phi \qquad \text{(in air)} \tag{7}$$

$$\mu\phi_{xx} + \phi_{zz} = \mu K^2 \phi \qquad \text{(in ferrite)}. \tag{8}$$

The boundary conditions to be satisfied are

1) $b_x = 0$ at $x = 0$ and a;
2) $b_z = 0$ at $z = 0$ and b;
3) ϕ being continuous at the interfaces $z = z_1$ and z_2;
4) b_z being continuous at the interfaces $z = z_1$ and z_2.

Here b_x and b_z are the components of $\bar{b}$ in the x and the z directions, respectively.

In this special case when the ferrite slab extends to both waveguide walls, it can be treated as a boundary value problem and mode analysis is employed effectively to solve for the dispersion characteristics for the different modes of propagation.

The following forms of ϕ in the three regions satisfy the boundary conditions 1 and 2 and can be expressed as

$$\phi_1 = \sum_{n=0}^{\infty} A_n \cos n\pi x/a \cosh\gamma_n'(b - z) e^{-jKy} \tag{9}$$

$$\phi_2 = \sum_{n=0}^{\infty} \left(\cos n\pi x/a - \frac{aK_1K}{\mu n\pi} \sin n\pi x/a\right) [B_n \cos\gamma_n z + C_n \sin\gamma_n z] e^{-jKy} \tag{10}$$

$$\phi_3 = \sum_{n=0}^{\infty} D_n \cos n\pi x/a \cosh\gamma_n' z e^{-jKy} \tag{11}$$

where A_n, B_n, C_n, and D_n are constants and γ_n' and γ_n are phase constants given by

$$\gamma_n' = \left[K^2 + (n\pi/a)^2\right]^{1/2}$$

$$\gamma_n = \alpha\gamma_n'$$

and

$$\alpha^2 = -\mu, \qquad \mu < 0.$$

The potential functions and normal magnetic fields in the air (eqs. (9) and (11)) and in YIG (eq. (10)) are matched at the interfaces $z = z_1$ and z_2 on the basis of conditions 3 and 4. This yields a set of coupled equations where the elimination of the unknown constants A_n, B_n, C_n, and D_n is necessary in order to find the dispersion relation. However, by using a certain procedure, A_n and D_n can be eliminated and the following systems of linear equations in B_n and C_n can be obtained:

$$B_m R_{mm} + C_m S_{mm} + P \sum_{n=0}^{\infty} (B_n R_{mn} + C_n S_{mn}) = 0 \qquad n \pm m = \text{odd} \tag{12}$$

and

$$B_m R'_{mm} + C_m S'_{mm} + P \sum_{n=0}^{\infty} (B_n R'_{mn} + C_n S'_{mn}) = 0 \qquad n \pm m = \text{odd} \tag{13}$$

where R_{mn}, S_{mn}, R'_{mn}, and S'_{mn} are known constants given by

$$R_{mn} = \beta_{mn}(\gamma_m' \tanh\gamma_m' s_2 \cos\gamma_n z_2 - \gamma_n \sin\gamma_n z_2) \tag{14a}$$

$$S_{mn} = \beta_{mn}(\gamma_m' \tanh\gamma_m' s_2 \sin\gamma_n z_2 + \gamma_n \cos\gamma_n z_2) \tag{14b}$$

$$R'_{mn} = \beta_{mn}(\gamma_m' \tanh\gamma_m' z_1 \cos\gamma_n z_1 + \gamma_n \sin\gamma_n z_1) \tag{14c}$$

$$S'_{mn} = \beta_{mn}(\gamma_m' \tanh\gamma_m' z_1 \sin\gamma_n z_1 - \gamma_n \cos\gamma_n z_1) \tag{14d}$$

with

$$\beta_{mn} = \begin{Bmatrix} 1, & m = n \\ 1/(n^2 - m^2), & m \neq n \end{Bmatrix}$$

$$s_2 = b - z_2$$

and

$$p = -4aK_1K/\mu\pi^2.$$

Note that all modes except the zeroth-order are coupled and one cannot exist without the others. This phenomenon leads to mode coupling between the propagating waves.

Equations (12) and (13) provide an infinite number of linear equations in B_n and C_n which can be expressed in the following matrix form:

$$\begin{bmatrix} M_{11} & PM_{12} & \cdot & PM_{1i} & \cdot \\ PM_{21} & M_{22} & \cdot & PM_{2i} & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ PM_{i1} & PM_{i2} & \cdot & M_{ii} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_i \\ \cdot \end{bmatrix} = 0 \tag{15}$$

where

$$M_{ij} = \begin{pmatrix} R_{ij} & S_{ij} \\ R'_{ij} & S'_{ij} \end{pmatrix} \qquad \text{for } i \pm j = \text{odd and for } i = j$$

$$M_{ij} = 0 \qquad \text{for } i \pm j = \text{even}$$

and

$$N_i = \begin{pmatrix} B_i \\ C_i \end{pmatrix}.$$

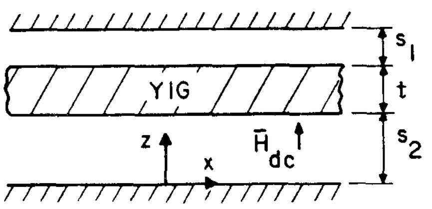

Fig. 2. YIG slab between two ground planes (normal field).

A nontrivial and unique solution of (15) for N_i's exists if the infinite determinant of the coefficient matrix is set to zero. For the purpose of numerical computations, which follow in the next section, the infinite determinant is truncated to a finite order.

A Degenerate Case

It is interesting to note that several degenerate cases of the formulation derived above reduce to problems that have been investigated previously [7].

As noted earlier, the zeroth-order mode is uncoupled and corresponds to the case when $a \to \infty$, i.e., when the YIG slab is placed between two ground planes. This structure, shown in Fig. 2, has been investigated extensively [8], [9].

To derive the dispersion relation from our formulations, the determinant of the zeroth-order mode, obtained from (12) and (13) (with $m = n = 0$), is set to zero as follows:

$$\begin{vmatrix} R_{00} & S_{00} \\ R'_{00} & S'_{00} \end{vmatrix} = 0 \Rightarrow R_{00}S'_{00} - R'_{00}S_{00} = 0 \qquad (16)$$

Note that in this special case $\gamma'_0 = K$ and $\gamma_0 = \alpha K$.

Upon substitution and simplification, (16) becomes

$$\tan \alpha K t = \frac{\alpha(\tanh K s_2 + \tanh K z_1)}{\alpha^2 - \tanh K s_2 \tanh K z_1} \qquad (17)$$

where t is the thickness of the YIG slab and is equal to $z_2 - z_1$.

Equation (17) was reported exactly by Daniel *et al.* [10].

III. Computer Simulation and Results

To obtain a nontrivial solution for higher order modes (other than zero) from the system of linear equations given by (15), the determinant of the coefficient matrix which is infinite in size must be zero. However, for practical purposes, the matrix was properly truncated for best accuracy. The truncation cutoff point of the matrix depends on the mode of propagation. For example, for the first- and second-order modes, the minimum matrix sizes were found to be 4×4 and 6×6, respectively. For higher order modes, larger matrices must be considered.

A computer program was written to find the determinant of the truncated coefficient matrix. With the aid of a proper computer algorithm, the determinant roots of the dispersion relation were found through several iterations. Through the results of these analyses it was found that the wave propagation is symmetrical in the guide cross section with respect to the slab position. Furthermore, in contrast gation is reciprocal for K and $-K$. Fig. 3 shows the effect of lowering the slab position. Besides the zeroth-order mode, two higher order modes are also shown. These higher order modes exist due to the finite width of the slab. In Fig. 4, the corresponding group time delays for different modes are plotted. The time delay increases as the slab is moved toward the center of the guide. Fig. 5

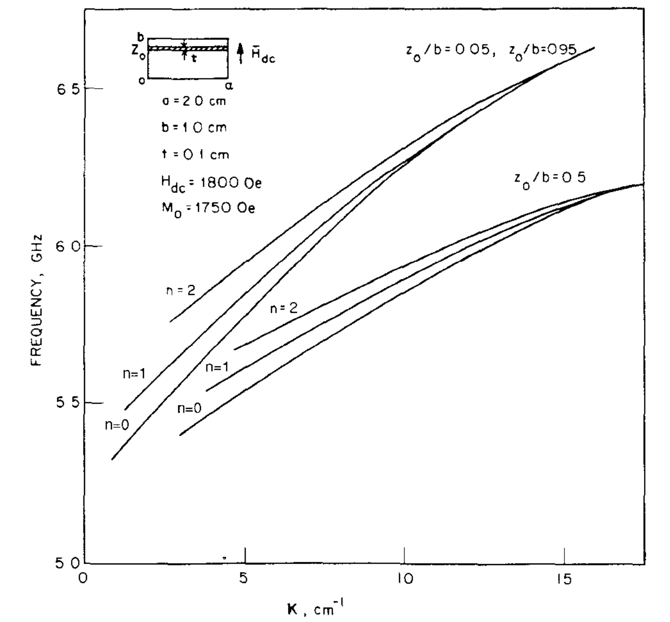

Fig. 3. Effect of slab position on the dispersion characteristics. Several modes of propagation are shown for each position of the slab.

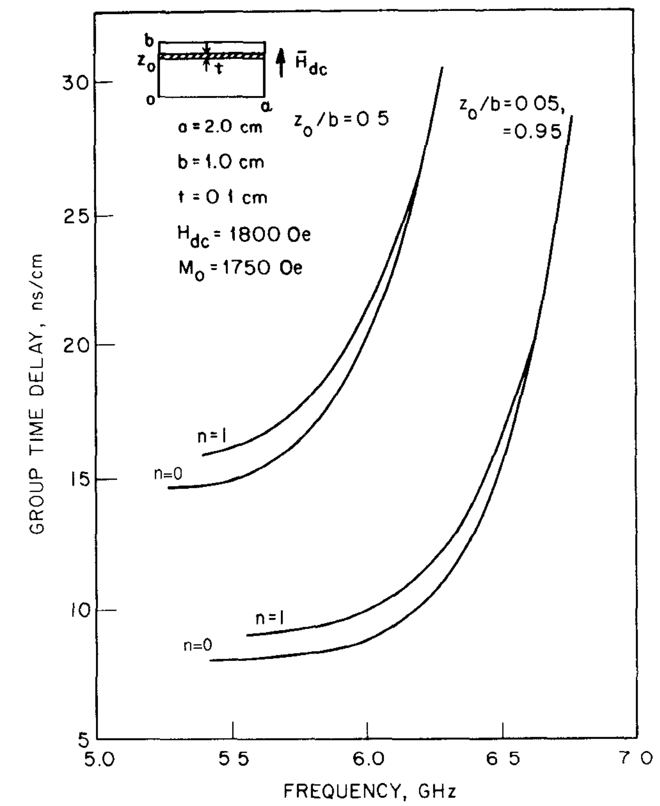

Fig. 4. Group time delay versus frequency.

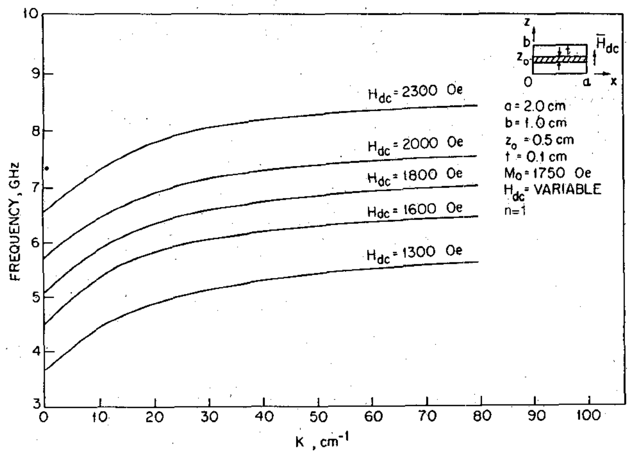

Fig. 5. Dispersion characteristics for various magnetic fields ($n = 1$).

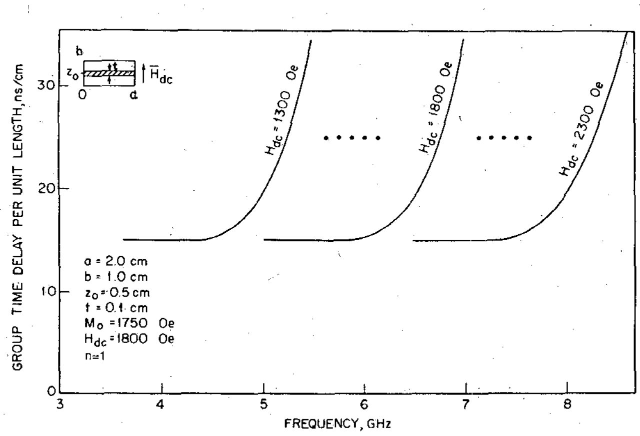

Fig. 6. Group time delay characteristics ($n = 1$).

shows the effect of normal magnetic field for several bias field values for the first-order mode. The dispersion curves are simply shifted to higher ranges of frequencies as the bias field value is increased. This effect on the group time-delay characteristics is shown in Fig. 6.

IV. Conclusions

The propagation of magnetostatic waves in a ferrite (such as YIG) slab inside a rectangular waveguide was analyzed. The employment of the mode analysis technique yielded the dispersion relations in terms of an infinite determinant. Using a proper truncation procedure, several important effects were studied. The dependence of the dispersion relation and group time delay per unit length on the position of the YIG slab and value of the bias field was presented.

From all these results, it becomes evident that in order to achieve high time delays, the slab should be positioned in the center of the guide, while for higher device bandwidths, the YIG slab should be positioned at the top or bottom of the guide. Thus there exists a tradeoff between the time delay per unit length and the device bandwidth, and some design compromises should be made. Finally, the tunable properties of the waveguide structure (Fig. 1) by means of a normal magnetic bias field were investigated and the results indicate that the waveguide structure can be tuned to any desired frequency range simply by shifting the bias magnetic field to a proper value.

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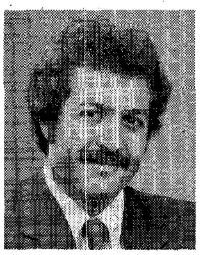

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